Çankaya University – ECE Department – ECE 376

Student Name :
Student Number :

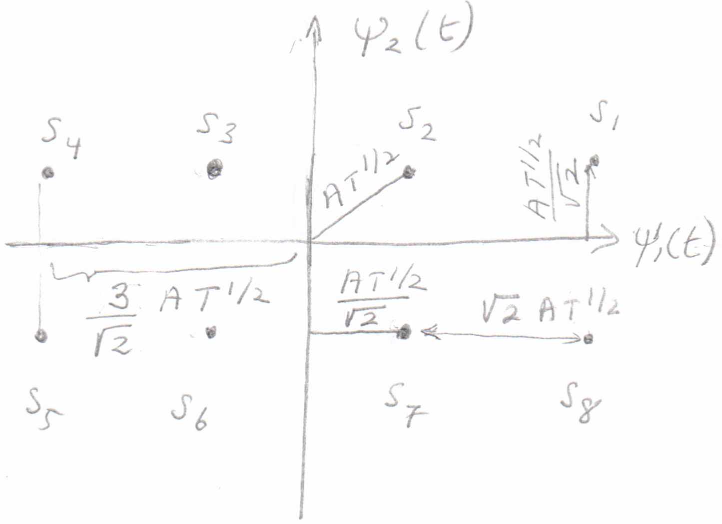
Open source exam
Duration : 2 hours

Questions

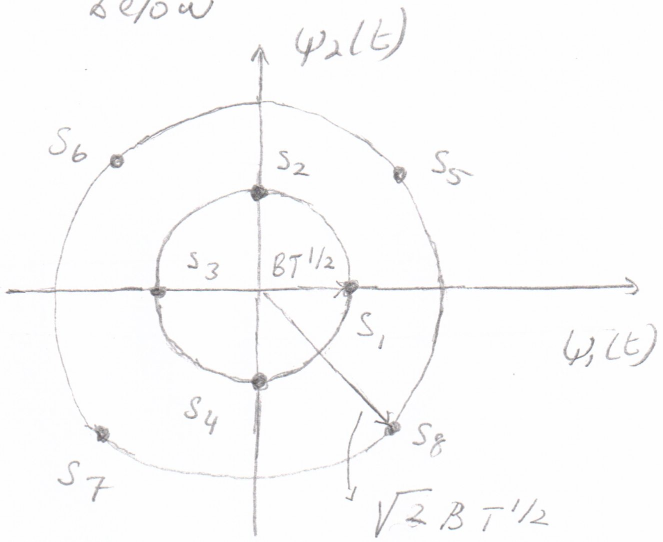
1. (70 Points) For QAM of $M = 8$, draw at least two possible constellation diagrams, related signal time waveforms and orthonormal basis functions. Write the mathematical expressions for the signal time waveforms and orthonormal basis functions. For this modulation type, draw the receiver diagram as correlator and matched filter (MF), find the output from the MF detector, if $s_4(t)$ was transmitted and compare this output to the output obtained from the correlator.

(sample)

Solution: For $M = 8$, two different QAM constellations are shown below

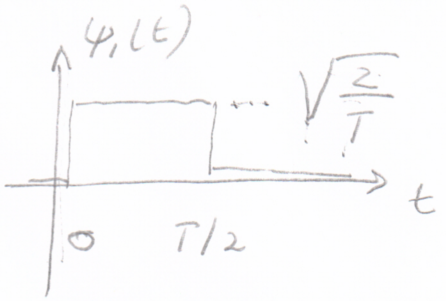


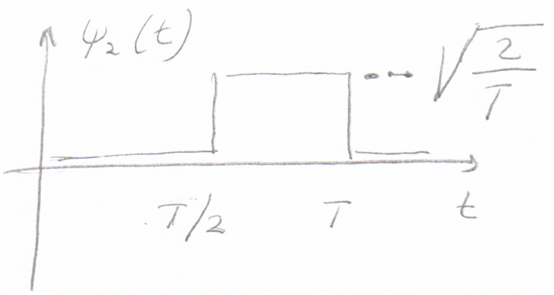
Constellation A



Constellation B

Basis functions for A and B are the same





ELE 376 MT- 29.03.2011 - Insert page (2)

Signal waveforms expressions for Constellation A

$s_1(t) = 3A(T/2)^{1/2}\varphi_1(t) + A(T/2)^{1/2}\varphi_2(t)$

or without $\varphi_1(t)$ and $\varphi_2(t)$

$s_1(t) = 3A \qquad 0<t<T/2$

$s_1(t) = A \qquad T/2<t<T$

$s_1 = [3A(T/2)^{1/2} \quad A(T/2)^{1/2}] \leftarrow$ as signal vector

$\mathcal{E}_{s_1} = 5A^2T \quad , \quad |s_1| = \sqrt{5}\, A T^{1/2}$

$s_2(t) = A(T/2)^{1/2}\varphi_1(t) + A(T/2)^{1/2}\varphi_2(t)$

$s_3(t) = -A(T/2)^{1/2}\varphi_1(t) + A(T/2)^{1/2}\varphi_2(t)$

$s_4(t) = -3A(T/2)^{1/2}\varphi_1(t) + A(T/2)^{1/2}\varphi_2(t)$

$s_5(t) = -3A(T/2)^{1/2}\varphi_1(t) - A(T/2)^{1/2}\varphi_2(t)$

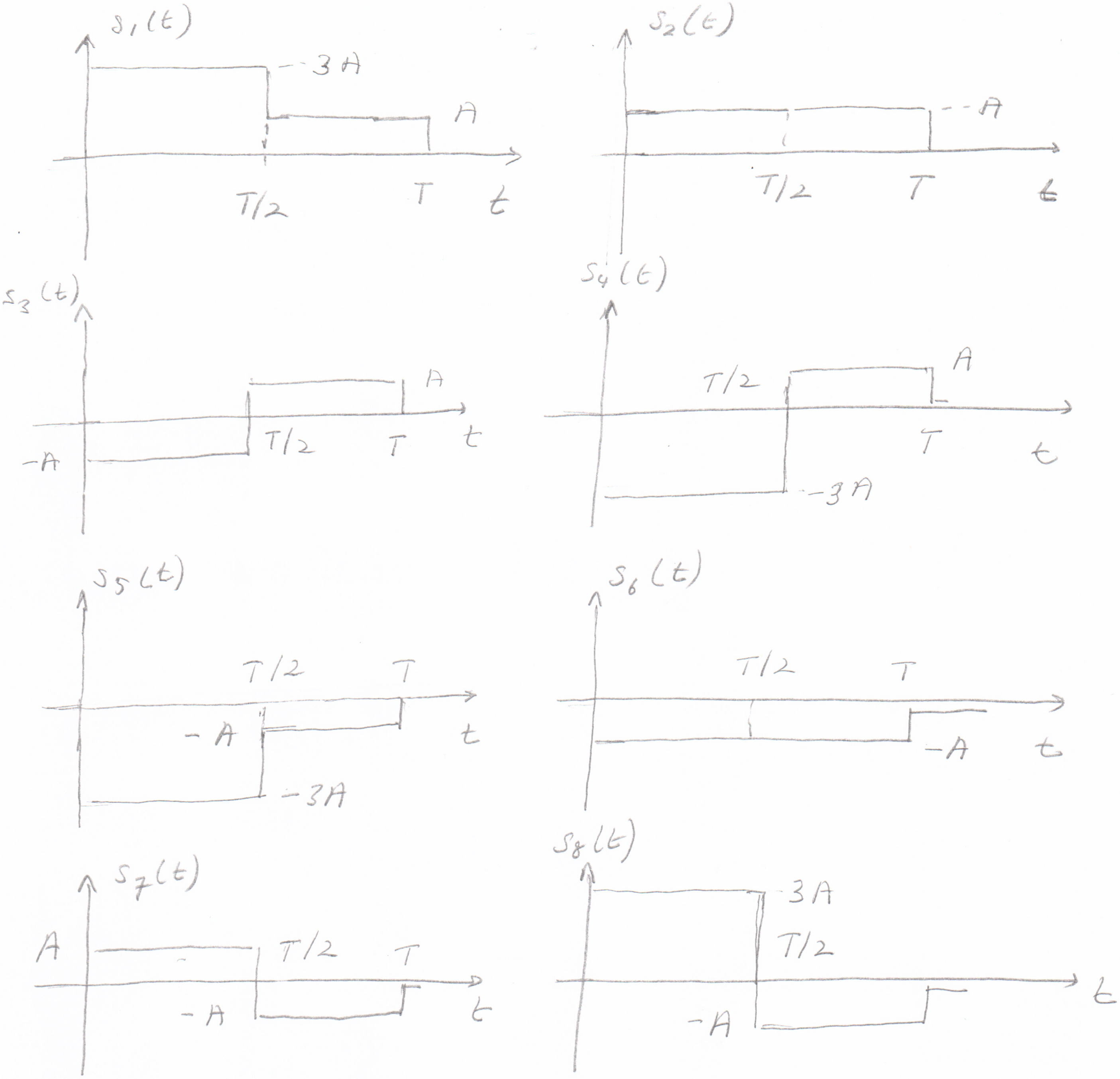
$s_6(t) = -A(T/2)^{1/2}\varphi_1(t) - A(T/2)^{1/2}\varphi_2(t)$

$s_7(t) = A(T/2)^{1/2}\varphi_1(t) - A(T/2)^{1/2}\varphi_2(t)$

$s_8(t) = 3A(T/2)^{1/2}\varphi_1(t) - A(T/2)^{1/2}\varphi_2(t)$

ECE 376 MT-29.03.2011-Insert page (2)

Signal waveform plots for Constellation A



$$\mathcal{E}_{s_1} = \mathcal{E}_{s_4} = \mathcal{E}_{s_5} = \mathcal{E}_{s_8} = 5A^2T$$

$$\mathcal{E}_{s_2} = \mathcal{E}_{s_3} = \mathcal{E}_{s_6} = \mathcal{E}_{s_7} = A^2T$$

ECE 376 MT- 29.03.2011 - Insert Page (3)

Signal waveform expressions for Constellation B

$s_1(t) = B\,T^{1/2}\,\psi_1(t)$

without basis functions

$s_1(t) = \sqrt{2}\,B \qquad 0 < t < T/2$

$\mathcal{E}_{s_1} = B^2 T \qquad |s_1| = B\,T^{1/2}$

$s_1 = [\,B\,T^{1/2} \quad 0\,]$ ← As signal vector

$s_2(t) = B\,T^{1/2}\,\psi_2(t)$

$s_3(t) = -B\,T^{1/2}\,\psi_1(t)$

$s_4(t) = -B\,T^{1/2}\,\psi_2(t)$

$s_5(t) = B\,T^{1/2}\,\psi_1(t) + B\,T^{1/2}\,\psi_2(t)$ or

$s_5(t) = \sqrt{2}\,B \qquad 0 < t < T$

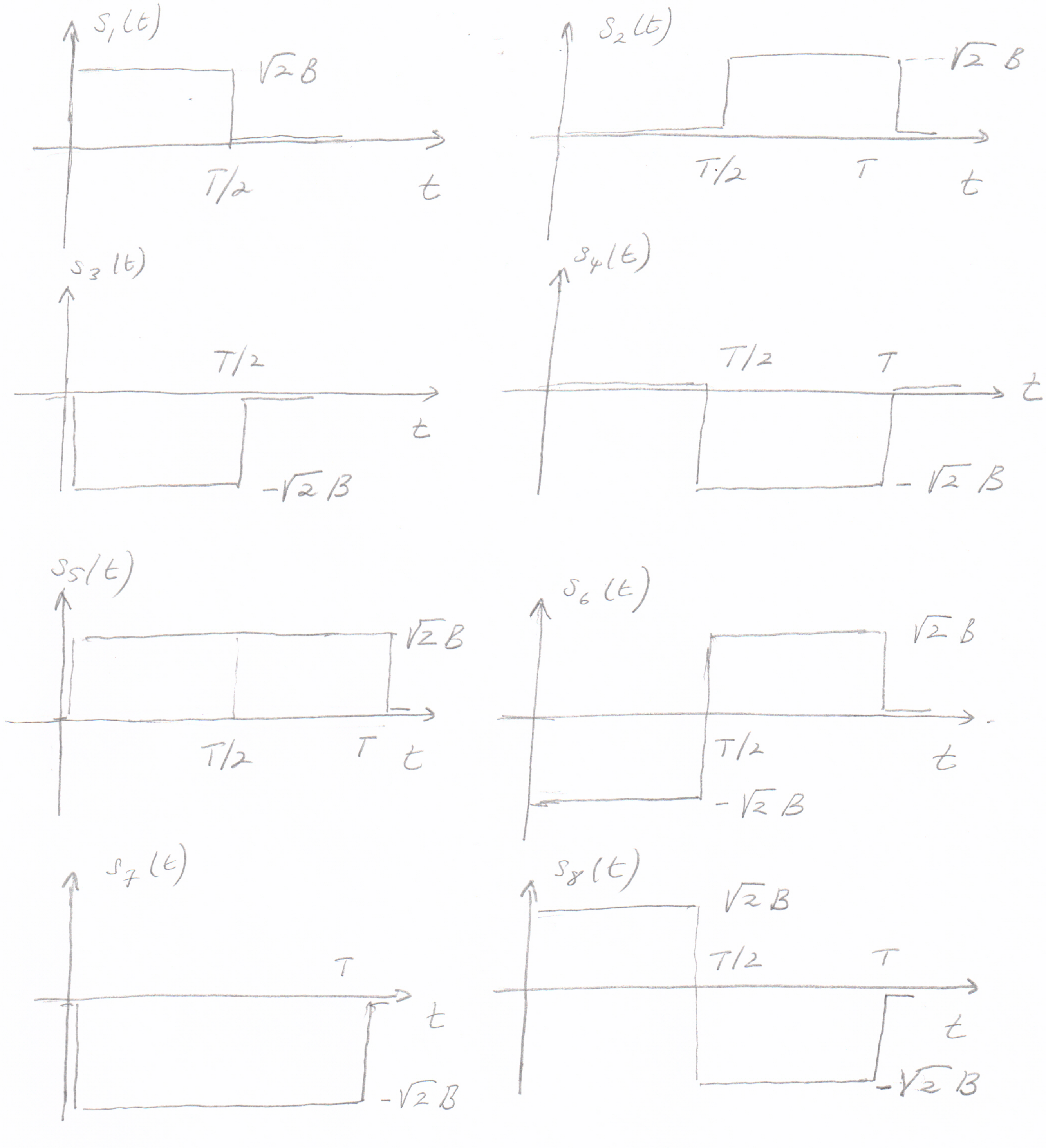
$s_6(t) = -B\,T^{1/2}\,\psi_1(t) + B\,T^{1/2}\,\psi_2(t)$

$s_7(t) = -B\,T^{1/2}\,\psi_1(t) - B\,T^{1/2}\,\psi_2(t)$

$s_8(t) = B\,T^{1/2}\,\psi_1(t) - B\,T^{1/2}\,\psi_2(t)$

ECE 376 MT – 29.03.2011 – Insert Page (4)

Signal waveform plots for Constellation B

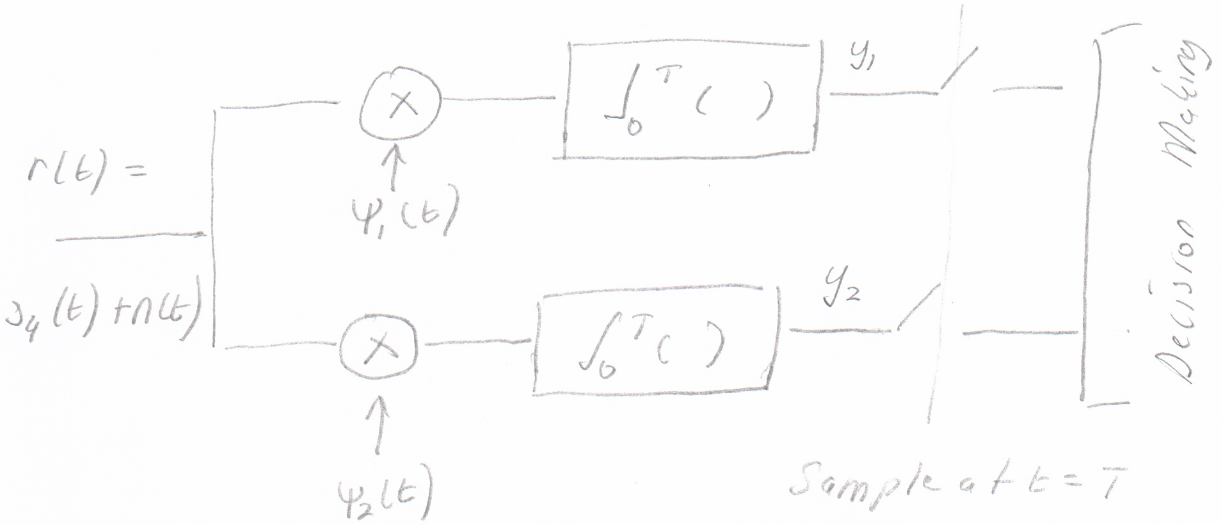


$$\mathcal{E}_{s_1} = \mathcal{E}_{s_2} = \mathcal{E}_{s_3} = \mathcal{E}_{s_4} = B^2 T$$

$$\mathcal{E}_{s_5} = \mathcal{E}_{s_6} = \mathcal{E}_{s_7} = \mathcal{E}_{s_8} = 2B^2 T$$

ECE 376 MT - 29.03.2011 - Insert Page (5)

For both constellations, receiver structure is the same since $N=2$ and $\varphi_1(t)$ and $\varphi_2(t)$ are identical, the correlator will be as follows



If $s_4(t)$ was transmitted from constellation A then

$$r(t) = s_4(t) + n(t)$$

not actually required

$$y_1(t) = \int_0^T [s_4(t) + n(t)]\,\varphi_1(t)\,dt$$

$$y_{1s}(t) = \int_0^T s_4(t)\,\varphi_1(t)\,dt = -3A\,(T/2)^{1/2}$$

$$y_{1n}(t) = \int_0^T n(t)\,\varphi_1(t) = n_1$$

ECE 376 MT - 29.03.2011 - Insert Page (6)

$$y_2(t) = \int_0^T [s_4(t) + n(t)]\, \varphi_2(t)\, dt$$

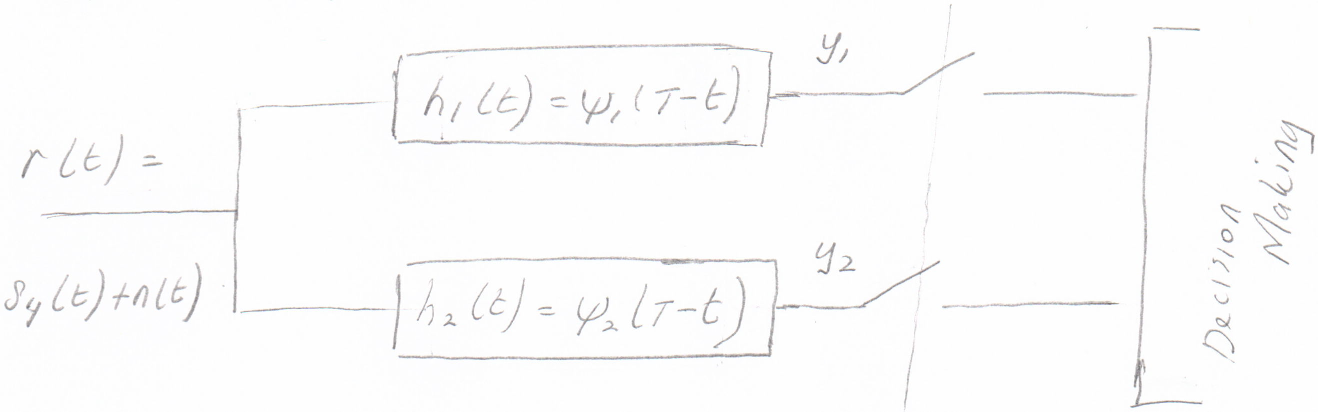
$$y_{2s}(t) = \int_0^T s_4(t)\, \varphi_2(t)\, dt = A(T/2)^{1/2}$$

$$y_{2n}(t) = \int_0^T n(t)\, \varphi_2(t)\, dt = n_2$$

Therefore r vector

$$r = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} -3A(T/2)^{1/2} + n_1 \\ A(T/2)^{1/2} + n_2 \end{bmatrix}$$

When MF is used



$$y_1(t) = \int_0^t [s_4(\tau) + n(\tau)]\, h_1(t-\tau)\, d\tau$$

at sampling instance of $t = T$

$$h_1(t-\tau) = \varphi_1(T-T+\tau) = \varphi(\tau)$$

$$y_1(t=T) = \int_0^T [s_4(\tau) + n(\tau)]\, \psi_1(\tau)\, d\tau$$

same as correlator

When $s_4(t)$ in constellation B is used

$$r = \begin{bmatrix} n_1 \\ \\ -BT^{1/2} + n_2 \end{bmatrix}$$

It is interesting to find that if we calculate $C(r, s_m)$ (correlation metrics) or $D(r, s_m)$ (distance metrics) using above r vectors of constellation A and B we should arrive (without noise) at the decision that $s_4(t)$ was transmitted.

HW: Test this for A and B constellations

ECE 376 MT- 29.03.2011 - Insert page (8)

Notes: 1) It is interesting to compare and establish equal energy (or power) basis for constellation A and B. For this (since M is the same)

Σ Total energy of signal vectors in constellation A

$\equiv$ Σ Total energy of signal vectors in constellation B

$$4 \times 5A^2T + 4 \times A^2T \equiv 4 \times B^2T + 4 \times 2B^2T$$

$$24A^2 = 12B^2 \quad \text{or} \quad A = \frac{B}{\sqrt{2}}$$

It is important to find this equivalence if we want to compare different constellations from Pe point of view

2) It is known from the literature that starting with $M = 8$, QAM has a better performance than PSK for the same M. [Ref: J. Proakis, Digital Communications,

ECE 376 MT-29.03.2011 - Insert Page (9)

Mc.Graw Hill, 2005, pp. 281-282]. But for this a constellation of QAM has to be found that has an increased min distance between signal vector ends provided that the compared 8 QAM and 8 PSK have the same energies.

HW: Suggest ways on how to find such a 8 QAM constellation.

2. (30 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones justify your answer.

a) For a given M, PSK and QAM have the same bandwidth requirement : True, since they use the same basis functions, thus dividing symbol interval T into two time slots of 0 − T/2 and T/2 → T.

b) For a given modulation type, as M increases, our bandwidth requirement increases as well : False, when M increases symbols begin to group more binary waveforms, thus bit rate increases and bandwidth requirement remains the same (since N remains the same)

c) ASK requires more bandwidth than PSK or FSK :

False since in ASK, N = 1 and in PSK or FSK N = 2 (except in antipodal PSK, where N = 1) Band width of ASK < Band width (PSK or FSK)

d) By increasing M, we increase symbol rate and also can transmit more bit rate :

The first part is false since when M increases symbol rate does not increase, but the second part is true, since increasing M means grouping more binary waveforms into one symbol.

e) By increasing N (dimensionality of signal space), we reduce the minimum distance between adjacent signal vectors :

False, we do the reverse. When signal vector lengths are kept the same (i.e their energies remain constant) but we increase N, then the distance between signal vector ends increase